THE ORGANIZED VORTEX STRUCTURE IN RELAXING GAS FLOW.

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Introduction

The flows of a weakly non-equilibrium compressible gas are satisfactorily described by the Navier – Stokes equations. The stress tensor in these equations contains two dissipative coefficients that are dynamic viscosity η_1 and bulk viscosity η_2. The last one takes into account relaxation of the excited internal degrees of freedom of gas molecules.

In paper [1] some experimental results on the bulk viscosity influence on the value of the critical Reynolds numbers of the laminar-turbulent transition (LTT) are presented. From the experimental data obtained it follows that, with increasing bulk viscosity, the value Re_c of LTT also grows. In this case, registered change reached ten percent. However, the matter of the reliability of the obtained data have been debatable. The calculations, which were derived within the framework of linear stability theory in papers [2, 3], have shown that bulk viscosity influence on LTT is rather little. In [2] the calculations were carried out for air at Mach number $M_0 = 4;5$ and relation of bulk viscosity to dynamic viscosity $\alpha = \eta_2 / \eta_1$ didn't exceed unit. In paper [3] the estimations were obtained up to $\alpha \le 30$.

At the same time the data of the experiments [1] correspond immediately to the nonlinear stage of transition in the turbulent regime at $\alpha \approx 7$. It's known from the modern scenarios of the transition and generation of turbulence are that these processes are essential nonlinear. They realize by means of origin, evolution and decay of certain vortex structures [4]. In boundary layers and pipes they are known λ-structures, in two-dimension free shear flows and jets they are two-dimensional vortices extended on the transversal coordinate. Under this interpretation the generation of turbulence can be considered as the process of LTT which is stochastically repeated in space and time. This fact allows one to assume that bulk viscosity influence on the development of perturbations at the nonlinear stage can be estimated by simulating the interaction of the solitary vortex structure with the basic (mean) flow. In this paper the simple model of the evolution of the spanwise vortex structure in a shear flow of non-equilibrium molecular gas is considered for such a purpose.

Flow model and statement of problem

The modeling problem is posed as follows. The gas shear flow is considered in a two-dimension square area with the side l. The unperturbed velocity field is

$$\vec{\tilde{U}} = 2U_0 \tilde{y} \vec{i} / l, \qquad \tilde{y} \in [-l/2; l/2].$$

Here $\vec{i}$ is a unit vector along axis $\tilde{x}$, U_0 is an absolute value of a velocity vector at the upper and lower bounds of a cell. The finite amplitude of the perturbation, which is a circle cross vortex with radius R_0 with uniform vorticity Ω_0 centered in the origin of coordinates is imposed on the basic flow at the moment $\tilde{t}_0$. Then the evolution of the structure in a compressible viscous flow of thermal exited gas is considered.

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Abstract

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Under an assumption of low excitation, the evolution of the structure in a modeling cell is described by the system of the Navier – Stokes equations of compressible gas. It is assumed that the transport coefficients are constant. In dimensionless variables

$$t=\tilde{t}/\tau_0, x=\tilde{x}/2R_0, y=\tilde{y}/2R_0, u_x=\tilde{u}_x/U_0, u_y=\tilde{u}_y/U_0,$$
$$\rho=\tilde{\rho}/\rho_0, T=\tilde{T}/T_0, p=\tilde{p}/\rho_0 U_0^2, \tau_0=2R_0/U_0$$

the system of the equations is written as follows

$$\begin{aligned}
&\frac{d\rho}{dt}+\rho\frac{\partial u_x}{\partial x}+\rho\frac{\partial u_y}{\partial y}=0,\\
&\rho\frac{du_x}{dt}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{4}{3}\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{1}{3}\frac{\partial^2 u_y}{\partial x\partial y}\right)+\frac{\alpha}{\mathrm{Re}}\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_y}{\partial x\partial y}\right),\\
&\rho\frac{du_y}{dt}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{4}{3}\frac{\partial^2 u_y}{\partial y^2}+\frac{1}{3}\frac{\partial^2 u_x}{\partial x\partial y}\right)+\frac{\alpha}{\mathrm{Re}}\left(\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_x}{\partial x\partial y}\right),\\
&\rho\frac{dT}{dt}=(\gamma-1)M_0^2\frac{dp}{dt}+\frac{1}{\mathrm{Re\,Pr}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{(\gamma-1)M_0^2}{\mathrm{Re}}\left[\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)^2+\right.\\
&\left.+2\left(\frac{\partial u_x}{\partial x}\right)^2+2\left(\frac{\partial u_y}{\partial y}\right)^2\right]+\frac{(\gamma-1)M_0^2(3\alpha-2)}{3\mathrm{Re}}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)^2,\\
&p=\gamma M_0^2\rho T, \frac{d}{dt}=\frac{\partial}{\partial t}+u_x\frac{\partial}{\partial x}+u_y\frac{\partial}{\partial y}.
\end{aligned}\tag{1}$$

Here $M_0=U_0/c_0$ is the Mach number, $\mathrm{Re}=2\rho_0 U_0 R_0/\eta_1$ is the Reynolds number, $\mathrm{Pr}=\eta_1 c_p/\lambda_0$ is the Prandtl number, c_0 is the adiabatic sonic speed, c_p and λ_0 are the specific heat at constant pressure and the conductivity respectively, and γ is the ratio of specific heats. The initial - boundary conditions for system (1) were chosen in the following form. At the initial moment, the components of the vector velocity $\vec{u}$, temperature T and density ρ are correspondingly equal

$$u_x=\begin{cases}\dfrac{2y}{\lambda}+\dfrac{\beta}{2}\left(\dfrac{y}{x^2+y^2}\right); x^2+y^2>0.25\\ \dfrac{2y}{\lambda}+2\beta y; x^2+y^2<0.25\end{cases}; u_y=\begin{cases}-\dfrac{\beta}{2}\left(\dfrac{x}{x^2+y^2}\right); x^2+y^2>0.25\\ -2\beta x; x^2+y^2<0.25\end{cases};$$
$$\rho=T=1.$$

Here the parameter $\beta=\Omega_0 R_0/2U_0$ characterizes the intensity of the imposed vortex perturbation, and the relation $\lambda=l/2R_0$ can be considered as an intermittence coefficient. The periodicity conditions were stated on the bounds of a modeling cell for normal velocity components, density and temperature. The conditions of antisymmetry were used for tangential velocity components.

The equation for integral production of turbulent energy in a modeling cell was used for estimation of bulk viscosity influence. It was deduced on the analogy with a similar equation for unbounded flows [5]. The velocity fluctuations were defined as the difference between instantaneous velocity values and velocity field of the carried gas flow

$$u'_x = u_x - \frac{2y}{\lambda}, u'_y = u_y$$

and the pulsations of density and pressure were defined by the relations

$$\rho' = \rho - 1, p' = p - \frac{1}{\gamma M_0^2}.$$

For fluctuating variables from system (1), the continuity and momentum equations were deduced, which are used to obtain the required equation for turbulent energy production by integration over the cell. It has the form

$$\frac{dE}{dt} = J_1 + J_2 - \frac{1}{\mathrm{Re}}\left(J_3 + \alpha J_4\right), \quad E = \frac{1}{\lambda^2} \int\limits_{-\lambda/2}^{\lambda/2} dx \int\limits_{-\lambda/2}^{\lambda/2} dy \frac{\rho \vec{u}'^2}{2}. \tag{2}$$

The integrals on the right-hand side of the Eq. (2) describe the interchange of energy between the structure and mean flow, the work under fluctuating compression and expansion of a gas, and the dissipation of perturbation energy, respectively. Let us notice that the integrals along the cell bounds arising during the derivation of Eq. (2) with used boundary conditions are precisely nullified. The values of the integrals J_1 and J_2 can be either positive or negative. The integrals J_3 and J_4 are positively defined. From Eq. (2) it follows that bulk viscosity brings in fluctuation energy production a term of the same sign as one caused by dynamic viscosity, and both terms reduce the energy of the perturbation. A similar equation was used for the energy estimation of the critical Reynolds number Re_c of LTT [6] which was obtained by minimization of the functional

$$\mathrm{Re} = \min\left(\frac{J_3 + \alpha J_4}{J_1 + J_2}\right)$$

at $dE/dt = 0$.

Therefore, it is possible to conclude that Re_c grows with an increase in the parameter α (or bulk viscosity η_2).

The method of the solution and the results of the calculations

For the numerical solving of the modeling problem an explicit – implicit finite – difference scheme with splitting on directions and physical processes was used [7]. The calculations were carried out for the Mach number $M_0 = 0.6$, the Prantdl number $\mathrm{Pr} = 1.0$ and the values of the parameter $\alpha = 0; 0.5; 1; 1.5; 2$. The characteristics of the considered structure were parametrized according to the data calculated for the mixing layer [8]. The Reynolds number, the intermittence coefficient, and the relative intensity of the vortex structure were chosen as $\mathrm{Re} = 60, \lambda = 3, \beta = 1.4$ respectively. The mesh domain contained $31 \times 31 = 961$ nodes with a step $h = 0.1$ on both coordinates, the time step was $\tau = 0.01$. The evolution was traced up to 600 time steps.

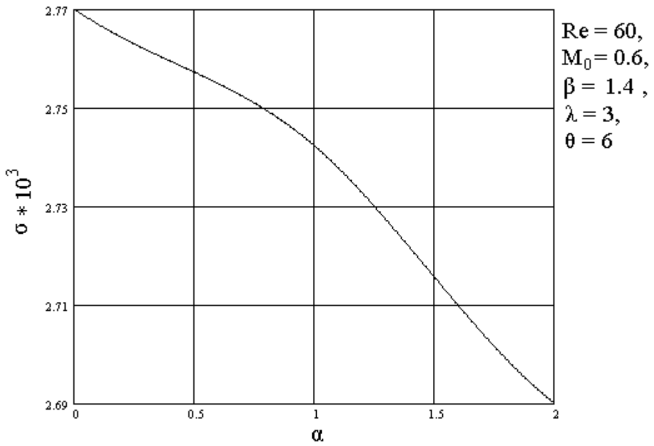

Figure 1.

To test the numerical scheme, diffusion of a circular isochoric Rankine vortex centered in the origin of coordinates in the absence of a shear flow were computed. Then the results was compared with the corresponding analytical solution. It was shown that the maximal relative error did not exceed 0.6% .

The average Reynolds stresses on a cell and on a time interval θ were calculated for the estimation of bulk viscosity influence on turbulence characteristics

$$\sigma = \frac{1}{\lambda^2 \theta} \left| \int\limits_{-\lambda/2}^{\lambda/2} dx \int\limits_{-\lambda/2}^{\lambda/2} dy \int\limits_{0}^{\theta} dt \rho u'_x u'_y \right| .$$

The dependences of the average Reynolds stresses on the parameter α (or bulk viscosity η_2) are presented in Fig. 1 for $\mathrm{Re} = 60, \lambda = 3, \beta = 1.4$ and $\theta = 6$. They show that average Reynolds stresses σ decrease if the value of bulk viscosity η_2 grows. The relative deviation in calculated interval α varies in the limits $0.4\% - 2.7\%$ compared with $\alpha = 0$.

As known, the organized vortices bring about 40% of contribution to the total Reynolds stresses in the mixing layer [9]. At the same time, viscous dissipation influence on large – scale

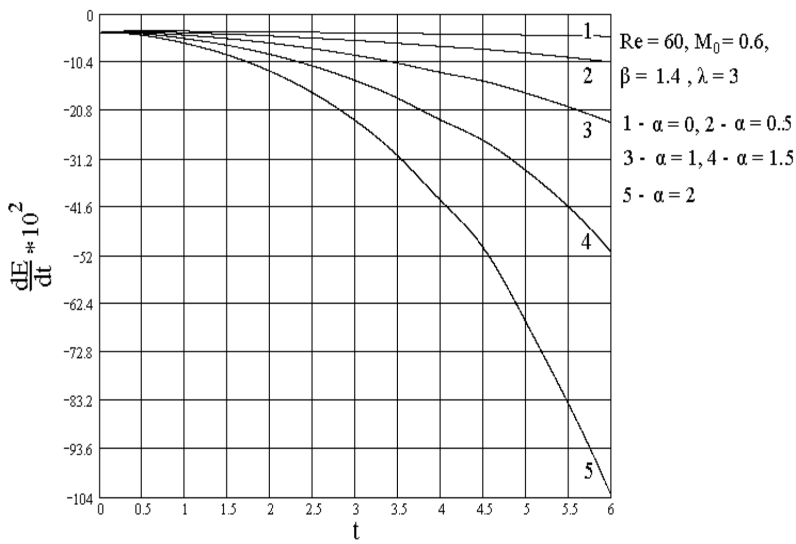

Figure 2

vortices is essentially less than the small-scale component of turbulence. This fact allow us to confirm that the decrease in the total Reynolds stresses can be estimated at least in the same limits for $0 \le \alpha \le 2$.

In Fig. 2 the time dependences of turbulent energy production for various values of the parameter α derived from Eq. (2) are plotted. Analyzing the curves, we see that the production of turbulent energy is always negative and decreases in this modeling problem. Moreover, the higher the value of bulk viscosity the more intense the suppression of fluctuations. The absence of the mechanism of the positive production of perturbation energy such as the stretching of quasi-streamwise vortices in the mixing layer [9] is one disadvantage of the model and it does not allow direct estimation of bulk viscosity influence on Re_c of LTT.

Conclusion

The results of numerical modeling of vortex structure dynamics in a shear flow allow us to conclude that bulk viscosity stabilizes LTT. It is shown that the Reynolds stresses averaged over space and time decrease if bulk viscosity increases with η_2 ranging from $\eta_2 = 0$ up to $\eta_2 = 2\eta_1$. The obtained change in the Reynolds stresses varies from 0.4% up to 2.7%. The production of turbulent energy in the considered modeling problem is always negative, and the higher the value α , the greater the decrease in turbulent fluctuations. The change is essential because by the order of the value it is comparative with the effect of practically employed mechanical devices for reducing turbulent drag, for example, by means of riblets [4]. Therefore, the hypothetical opportunity of processing turbulent drag in compressible flows of molecular gases by changing bulk viscosity has appeared. In connection with that, it is necessary to continue researches on the basis of more improved models.

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